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polar atoms, and not of ions, and to use the term ions only in its historical sense, that is to designate particles which migrate in the electrical field.

¹ Noyes and MacInnes, these PROCEEDINGS, **5**, 1919; *J. Amer. Chem. Soc.*, **42**, 1920 (239).

² *London, Phil. Mag.*, Ser. 6, **14**, 1907 (3).

³ *Ibid.*, **35**, 1918 (214, 354).

⁴ *J. Chem. Soc.*, **113**, 1918 (449, 627).

⁵ Bjerrum, *Zs. Elektrochem.*, **24**, 1918 (321).

⁶ *J. Amer. Chem. Soc.*, **33**, 1911 (1807, 1827, 1836).

⁷ *Ibid.*, **41**, 1919 (1951); **34**, 1912 (1631).

⁸ *Ibid.*, **33**, 1911 (1864).

ON A CONDITION FOR HELMHOLTZ'S EQUATION SIMILAR TO LAMÉ'S

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During the last thirty years the writer has been very much interested in the diffraction of sound, a subject suggested to him for theoretical treatment in 1888 by his teacher, the great von Helmholtz. Considering the great amount of paper spoiled in futile attempts to further the subject, the pessimistic view of Lord Rayleigh, and the amount of experimental results obtained by the writer, but not published, it seems proper, in accordance with a policy recently announced by the writer, to publish whatever he has in storage, however modest. A small attempt was made in a paper "On the Wave Potential of a Circular Line of Sources (*Proc. Amer. Acad. Arts Sci.*, December, 1911), an improvement on which has been recently made. The following paper is taken from a drawer, endorsed February 20, 1908, and like the other was written in the attempt to advance the theory of the megaphone.

The condition obtained by Lamé that a singly infinite family of surfaces shall be the equipotentials for some distribution is well known. It occurred to me to examine the condition that there may be a function V satisfying the differential equation investigated by von Helmholtz,

$$\Delta V + k^2 V = 0, \quad (1)$$

which comes from the wave equation

$$\frac{\partial^2 \varphi}{\partial t^2} = a^2 \Delta \varphi, \quad (2)$$

when we assume that φ contains a simple harmonic function of the time, the function V depending upon a single parameter q . If this is the case we have

$$\frac{dV}{dq} \Delta q + \frac{d^2V}{dq^2} h_q^2 + k^2 V = 0, \quad (3)$$

where Δ_q stands for the Laplacian and h_q for the gradient of q . Dividing by dV/dq we have

$$\Delta q + h_q^2 \frac{d(\log dV/dq)}{dq} + \frac{k^2}{d(\log V)/dq} = 0. \quad (4)$$

Since V and its derivatives depend on q alone, we may write this

$$\Delta q + h_q^2 \varphi(q) + \frac{k^2}{f(q)} = 0 \quad (5)$$

where

$$\begin{aligned} f(q) &= \frac{d(\log V)}{dq}, \quad \log V = \int f(q) dq + C, \quad V = A e^{\int f(q) dq} \\ \frac{dV}{dq} &= A e^{\int f(q) dq} f(q), \quad \log \frac{dV}{dq} = \log A + \int f(q) dq + \log f(q) \\ \varphi(q) &= d \frac{\left(\log \frac{dV}{dq} \right)}{dq} = f(q) + \frac{f'(q)}{f(q)}. \end{aligned}$$

Thus we may write (5).

$$\Delta q + h_q^2 \left\{ f(q) + \frac{f'(q)}{f(q)} \right\} + \frac{k^2}{f(q)} = 0, \quad (6)$$

which is the required condition. This may be put in another form as follows: Multiplying by $f(q)/h_q^2$ we have

$$\frac{df}{dq} + f^2 + \frac{f \Delta q + k^2}{h_q^2} = 0. \quad (7)$$

Now although it is not necessary for either Δq or h_q^2 to be functions of q alone, since the first two terms of (7) are such, the third term must be, say

$$\frac{f \Delta q + k^2}{h_q^2} = Q(q) f(q). \quad (8)$$

The equation

$$\frac{df}{dq} + f^2 + Qf = 0, \quad (9)$$

is not linear, but may be made so by substituting for f its reciprocal R . We then have

$$\frac{dR}{dq} - QR = 1 \quad (10)$$

of which the solution is

$$R = \frac{1}{f} = e^{\int Q dq} (B + \int e^{-\int Q dq} dq). \quad (11)$$

From (11) and (8) we accordingly obtain

$$\Delta q = Q h_q^2 - k^2 R = Q h_q^2 - k^2 e^{\int Q dq} (B + \int e^{-\int Q dq} dq), \quad (12)$$

which is the desired condition, containing one arbitrary constant and one arbitrary function, and reducing to Lamé's condition when $k = 0$.

It is easy to show that (12) is satisfied by a set of parallel planes, concentric spheres, or concentric circular cylinders, all these being parallel surfaces. It is not satisfied, as is the case with Laplace's equation, by a set of confocal ellipsoids. It was owing to the impossibility of finding other sets of surfaces to test the result, together with compunctions about Huygens' principle, that caused the writer to hold back the result. Recently, however, Somiliana has published a proof that these are the only sets of parallel surfaces that allow propagation according to Huygens' principle (*Atti Torino*, 54, (974-979). *Sulle relazioni fra il principio di Huygens e l'ottica geometrica*) in which he is obliged to use the formulae of differential geometry. I therefore venture to publish the above more simple result.

THE ABSORPTION OF X-RAYS BY CHEMICAL ELEMENTS OF HIGH ATOMIC NUMBERS

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Introduction.—A critical absorption wave-length characteristic of a chemical element is a wave-length such that the element absorbs X-rays longer than the critical value less than it does X-rays shorter than that value. Each chemical element has one critical absorption wave-length associated with its K series of characteristic emission lines. Duane and Hu¹ have shown that in the K series of rhodium the critical absorption wave-length is about one-fourth of one per cent shorter than that of the shortest line (the γ line) in the K emission series. The authors² found that the critical absorption in the K series of tungsten has a wave-length of about one-half of one per cent shorter than that of the $K\gamma$ emission line. Since the other characteristic X-ray series have longer wave-lengths than those of the K series, the K critical absorption wave-length is the shortest X-ray wave-length now known to be characteristic of a chemical element.

In the research described in this note the authors have measured the critical absorption wave-lengths in the K series of most of the available chemical elements from tungsten to uranium, both inclusive. They used an ionization spectrometer, and examined spectra of the first, second and third orders. In 1918 Duane and Shimizu³ measured four of these wave-lengths in spectra of the first order by the ionization method. Measurements with the same spectrometer of the K critical absorption had previously been made for most of the chemical elements down to man-